JOURNAL OF PROPULSION AND POWER
Vol. 26, No. 1, January–February 2010

Optimization of System Parameters for Liquid Rocket Engines with Gas-Generator Cycles

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DOI: 10.2514/1.40649

System design of liquid rocket engines must consider engine performance, weight, cost, and reliability requirements. A general design optimization framework has been developed in this paper to select the best system parameters for liquid rocket engines with gas-generator cycles. The object is to maximize the specific impulse and vacuum thrust-to-weight ratio of the engine with given system requirements and design assumptions by changing thrust-chamber pressure and mixture ratio. The system analysis, along with the engine weight estimation, is based on a modular scheme. Multidisciplinary design optimization formulations including multidisciplinary feasible and collaborative optimization are used, evaluated, and compared during the optimization process. Several techniques of multi-objective processing are also used to identify the Pareto frontier and the optimal compromise solutions. A proposed cryogenic-propellant engine using liquid oxygen and hydrogen with a gas-generator cycle is studied as a specific example. Moreover, uncertainties in the engine operation, such as thrust-chamber pressure and mixture ratio, are taken into account as random variables in the reliability-based optimization. Results are presented to illustrate the tradeoff between the engine performance and reliability requirements.

I. Introduction

LIQUID rocket engines (LREs) with gas-generator cycles have been widely used in the field of space propulsion, due to their relatively simple design, low development costs, high reliability, and mature technology. Furthermore, they offer the possibility of varying the thrust and propellant mixture ratio over a wide range as an open cycle. These rocket engines consist of a number of common components, such as the thrust chamber, gas generators, turbines, pumps, pipelines, and valves. With few exceptions, system design of such LREs must consider engine performance, weight, cost, and reliability, which usually pose conflicting requirements. Optimization studies for evaluating "what-if" scenarios and tradeoff studies are thus conducted by the engine designers to select the best values of engine parameters at the system level (including thrust-chamber pressure, nozzle expansion ratio, and propellant mixture ratio) to improve the LRE system performance while satisfying the launch vehicle requirements.

Most of today's engine design and optimization can be performed using computer programs, which are usually specific to a particular design organization and a certain category of application. These programs range in fidelity and scope, from conceptual system-level tools to high-fidelity computational fluid dynamics simulations. Typically, the system-level tools use a modular analysis approach, where individual engine components are modeled independently using thermodynamic and other appropriate relationships, and these separate modules are then integrated into a desired engine cycle by means of conservation laws to be analyzed. Binder [1] introduced an industry-standard LRE power balance tool, which keeps engine component modules as functions in a standard library. Goertz [2] proposed a modular procedure for system analysis of arbitrary LRE cycles with different propellant combinations at the operating state point. Manski et al. [3] developed a combined vehicle/propulsion analysis tool and applied it to investigate the tradeoff between engine performance and vehicle mass for single-stage-to-orbit feasibility studies. To provide "quick-look" answers to propulsion system trade studies for spacecraft or launch vehicle designers, Way and Olds [4] created a Web-based code to simulate LRE combustion with efficiencies for performance prediction. This model was updated, especially with respect to the chemistry model, by Bradford [5] to a commercial version. Cormier [6] and St. Germain [7] further developed a powerhead analysis code to provide a conceptual-level LRE analysis tool for launch vehicle design and optimization in a timely fashion. Similarly, Bradford et al. [8] produced a commercially available propulsion modeling tool for use in the conceptual and preliminary design of space transportation systems using LRE. Li et al. [9] also investigated the modular simulation of LRE systems incorporated in an expandable software package for the evaluation of engine configurations. Recently, Isselhorst [10] reported a software kernel capable of time-dependent simulation of the propulsion system for the launch vehicle and stage analysis. To the authors' knowledge, however, there has been little research on optimization of LRE system parameters in the preliminary design stage from the viewpoint of multidisciplinary design optimization (MDO).

In this paper, a design optimization framework has been developed to select best values of system parameters for LREs with gas-generator cycles. The optimization is aimed at maximizing the specific impulse and vacuum thrust-to-weight ratio of an engine with given mission requirements and design assumptions by changing thrust-chamber pressure and mixture ratio. MDO formulations including multidisciplinary feasible (MDF) and collaborative optimization (CO) are used, evaluated, and compared during the optimization process, in which an optimization algorithm combining the genetic algorithm and the gradient-based algorithm is employed to achieve the global optimum solution. The MDO approaches are

Presented as Paper 3743 at the 41st AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, Tucson, AZ, 10–13 July 2005; received 28 August 2008; revision received 6 October 2009; accepted for publication 6 October 2009.

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evaluated by applying them to a test problem: the system parameter optimization for a proposed hydrogen–oxygen rocket engine with a gas-generator cycle.

There are many techniques dealing with multi-objective problems in tradeoff studies [11–13]. Designers usually have preferences according to their own experience. Therefore, several methods of multi-objective processing are investigated for the aforementioned test problem, to obtain the Pareto frontier of two optimization objectives. Then the optimal compromise solutions are identified. All the results are compared against each other to show the impact of the different methods on the design variables and objectives. The Pareto frontier and optimal compromise solutions presented herein may provide a reference for further engine designs.

Finally, uncertainties in the engine operation, such as thrust-chamber pressure and mixture ratio, are also given preliminary consideration. The reliability-based optimization is implemented for a test case to obtain a more reliable LRE system design with the offset of a slight decrease in engine specific impulse.

II. LRE System Analysis

LRE system analysis is the foundation of the optimization of system parameters, which provides the values for design variables and parameters for design optimization through the thermodynamic calculations, stationary performance analysis, and engine weight estimation. All the codes are written in the object-oriented C++ programming language and structured to use input files wherever possible so as to speed up interchange between new components, save compile time, and facilitate the concurrent distributed processing.

A. Performance Analysis

In the thermodynamic calculation, the combustion product thermodynamic properties (which include the combustion gas temperature, specific heat, and gas constant) are calculated by a legacy code [14] using the minimization of free-energy method, given the composition of the propellants, chamber pressure, and oxidizer-to-fuel mixture ratio. The chemical properties for any of the generic propellant types in the C-H-O-N system have been built into this code.

Stationary performance analysis is the key to the simulation of the LRE system. A modular approach is employed for the quantitative analysis of the LRE feed system with gas-generator cycles. In the component library (including the thrust chamber, gas generators, turbines, pumps, pipelines, and valves), the stationary macroscopic behavior (typically including the pressure, temperature, flow rate, and power) of each module is simulated by the basic zero-dimensional analytical model with some empirical correlations [15]. These models are then integrated into an overall engine framework by means of conservation laws (i.e., the balance of flow, pressure, and power). Reference [16] gives the details on such system modeling. The simulation process for the present optimization study will be outlined in Sec. III.B. For the engine design mode, a required thrust at a selected ambient condition should be specified in the simulation. The other input parameters include 1) thrust-chamber pressure and ambient pressure at design altitude; 2) thrust-chamber and gas-generator mixture ratios; 3) pump rotational speed and inlet pressure; 4) turbine pressure ratio and efficiency; and 5) valve, injector, and heat-exchanger pressure-drop coefficients. Based on the data from the stationary characteristics calculation along with thermodynamic analysis, the LRE system performance can be determined for evaluation.

B. Weight Model

The launch capability and cost of vehicles are significantly affected by the LRE weight. Therefore, the engine weight should be included in the LRE system evaluation. It is usually difficult to evaluate the engine weight exactly in the preliminary design phase, however, because the engine geometry is unknown, and the engine weight can be affected by many other factors [7,8,17]. A series of approximate weight analysis formulations developed by Zhu et al. [17] are employed in the present study with some slight modifications. The weight model is based on parameter analysis of some existing similar rocket engines and is suitable for weight estimation for LREs with gas-generator cycles. Using this approach, the engine weight is determined by summing up the weights of the thrust chamber, gas generators, valves, turbopumps, and other miscellaneous components. The model can be expressed by the following equations, where a, b, c, and d are the corresponding fitting coefficients.

Thrust-chamber weight:

$$M_t = a \cdot F_c^b \cdot r_c^c \cdot P_c^d \tag{1}$$

where F_c is the thrust-chamber thrust in vacuum, r_c is the thrust-chamber mixture ratio, and P_c is the thrust-chamber pressure.

Gas-generator weight:

$$M_g = a \cdot \dot{m}_g^b \cdot P_3^c \tag{2}$$

where $\dot{m}_g$ is the mass flow rate of the gas generator, and P_3 is the gas-generator pressure.

Valve weight:

$$M_v = a \cdot \dot{m}_v^b \cdot P_v^c \tag{3}$$

where $\dot{m}_v$ is the mass flow rate through the valve, and P_v is the valve inlet pressure.

Turbopump weight:

$$M_p = a \cdot \dot{m}_p^b \cdot f_p^c \cdot H_p^d \tag{4}$$

where $\dot{m}_p$ is the mass flow rate of the pump, f_p is the pump rotational speed, and H_p is the pump's head rise.

Weight of other miscellaneous components:

$$M_l = a \cdot F_v^b \cdot P_3^c \tag{5}$$

where F_v is the engine thrust in vacuum.

The weight of each component above is approximated by related factors and component-specific coefficients for which the values are curve-fit approximations to historical data on existing similar rocket engines. Since most data on individual engine component weights is proprietary, Table 1 only lists the total engine weight estimates for the YF-73 and YF-75 engines, which are both gas-generator-cycle engines, in comparison with those data published in [18]. The precision of this model is acceptable for the design optimization of the LRE system in the preliminary stage of design.

III. Deterministic Optimization of System Parameters

A. Optimization Problem Statement

For the LRE system design, optimization studies are used to select the best system parameters while satisfying the propulsion system requirements, which means that the design space should receive more attention than the objective from the viewpoint of optimization in this case. Theoretically, all the input parameters for LRE system analysis, described in Sec. II.A, could be set as design variables. Engine designers, however, usually show more interest in those parameters that have primary impact on the LRE system analysis and optimization. In the present study, the optimization design variables are the thrust-chamber pressure P_c and mixture ratio r_c. The other input parameters are considered fixed in the optimization process. To improve the launch capability of rockets, it is useful for engine designers to increase the engine specific impulse and to decrease the weight of the engine. Therefore, the optimization objectives are

Table 1 Weight model verification

Case	Weight model results	[18] data	% error
YF-73 engine	222.0 kg	236 kg	5.9
YF-75 engine	585.2 kg	550 kg	6.4

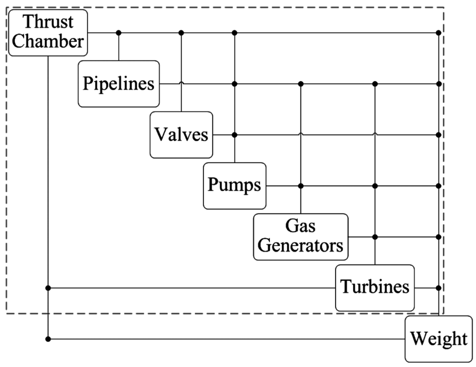

Fig. 1 Design structure matrix for LRE system with gas-generator cycles.

decided as the maximization of engine specific impulse I_e and vacuum thrust-to-weight ratio N_e. Meanwhile, boundaries of design variables and objectives are set as constraints for the optimization.

B. Design Structure Matrix

The design process of LRE system optimization involves the analytical modules of the thrust chamber, pipelines, valves, pumps, gas generators, turbines, and engine weight estimation. The interactions among these command-line modules can be conveniently represented by the design structure matrix (DSM), which is useful as an MDO technique for multidisciplinary or multisubsystem analysis. Figure 1 depicts the DSM for the optimization process in the present study. In the DSM, each box represents a specific disciplinary or subsystem analysis. Output from a process is shown as a horizontal line that exits a box, and input into a process is shown as a vertical line that enters a box. The offdiagonal circles that connect the horizontal and vertical lines represent couplings between the two processes. Circles in the upper triangle of the DSM represent feedforward couplings, and circles in the lower triangle of the matrix represent feedback couplings.

In Fig. 1, the part inside the dashed line represents the stationary performance analysis of an LRE system with gas-generator cycles, where the thrust-chamber thrust and the turbine-exhaust-gas thrust are provided by the thrust chamber and the turbines' simulation codes, respectively. Note that there are both feedforward and feedback circles, which means that the performance analysis process will be iterative until the sum of the above two thrusts approaches that required thrust. Once the specific impulse and the associated engine weight are finally determined in the simulation process, the optimizer can calculate the value of the objective function for the LRE system optimization.

C. Application of Design Framework Package

The typical optimization design process is known as the design–evaluate–redesign cycle, which involves the iteration of processing input files, running analysis and simulation codes, and analyzing output files. To enable such process integration and automation for the present study, the design process with all the LRE system analysis codes is defined as control and data flow in a commercially available design framework package [19]. Specific parameters and variables for optimization studies are mapped from the input and output files of the analysis codes through the graphical user interface of this framework. Then the LRE optimization design problem is set up in terms of design variables, objectives, constraints, and the initial starting point. By using the optimization algorithms and other techniques incorporated within the framework, the design cycle can be driven automatically until the design optimization criteria are satisfied, and thus the best design solution of LRE system parameters is identified.

Two MDO formulations, MDF and CO, are attempted in the present work. The following are the detailed methods, techniques, and results.

D. Multidisciplinary Feasible Method

The MDF formulation is the most basic of MDO approaches and has wide industry acceptance. Generally speaking, it is easier to find the optimal solution using MDF as the conventional approach to solve small-scale MDO problems, such as the present LRE system design optimization. In addition, the MDF solution is usually considered as the baseline result to evaluate other MDO formulations and their modifications. In the MDF formulation, an optimizer is imposed over the complete multidisciplinary or multisubsystem analysis, and system feasibility is maintained at each iteration of the optimization procedure, which means the analysis will be executed iteratively if there are strong couplings between the disciplines or subsystems.

Based on the previous procedure of the LRE system analysis, the MDF formulation can be stated as follows:

$$
\begin{aligned}
&\text{Find } P_c, r_c \\
&\min\ -\omega_1 I_e/A - \omega_2 N_e/B \\
&\text{subject to } I_e \geq I_{e0} \\
&F_0 \leq F \leq F_1 \\
&r_{e0} \leq r_e \leq r_{e1}
\end{aligned}
\tag{6}
$$

where the objective function is the weighted sum of the engine specific impulse and the vacuum thrust-to-weight ratio.

To find the global optimal solution, a type of hybrid algorithm, which combines the genetic algorithm (GA) and sequential quadratic programming (SQP), is used for the MDF formulation. The GA is initially applied to conduct an overall search for the design space of LRE system parameters to identify regions in which the best solutions might lie. Then the SQP is applied, starting from the solution obtained from the exploratory search, to conduct a more local search to identify the best solution in the region of interest. Such hybrid optimization strategy, maintaining advantages of the exploratory and numerical techniques, is widely used in aerospace applications [20,21].

As a specific example, a cryogenic-propellant rocket engine using liquid oxygen and hydrogen with a gas-generator cycle is studied. The weights of the two objectives are both set to 0.5. The corresponding MDF results are listed in Table 2.

E. Collaborative Optimization

The CO formulation is a bilevel MDO approach, which has both system- and local-level optimization based on the decomposition of the problem along the lines of the constituent disciplines or subsystems. This formulation, developed from the individual discipline feasible method, was originally introduced by Kroo et al. [22]. In the CO formulation, customary disciplinary or subsystem groupings are preserved and the concurrent distributed processing for the subsystem analyses and optimizations is also allowed. For the purpose of coordinating between subsystems and arriving at an

Table 2 Optimization solution from MDF formulation

	Design variables		Parameters								Objectives	
Case	P_c, MPa	r_c	P_f, MPa	P_o, MPa	P_3, MPa	F_1, kN	F_c, kN	F_t, kN	q_1 kg/s	q_3 kg/s	I_e s	N_e N/kg
Initial values	11.0	6.0	18.2	15.4	9.4	665.0	696.6	1.9	157.8	8.7	428.1	607.8
Optimization values	8.0	5.3	13.2	11.2	6.8	664.5	698.7	1.3	156.8	6.1	439.4	670.9

optimum MDO solution, the CO formulation creates copies of all the interdisciplinary coupling variables as additional design variables at the system level. The system optimizer then uses these copies to send out design targets to each subsystem. Instead of requiring multidisciplinary feasibility at each iteration of optimization, the system-level feasibility is to be achieved via the compatibility constraints, which are designed to drive the discrepancies between interdisciplinary coupling variables and corresponding system values to zero. Therefore, each iteration is feasible with respect to subsystem analyses but is not multidisciplinary feasible until an optimization solution is reached using the CO formulation.

In the present study, in order to examine the ability of the CO formulation to solve LRE system design problems, all the analysis codes for the gas-generator-cycle engine system are grouped into three modules according to the main rocket engine components: thrust-chamber module, gas-generator module, and turbopump module, which are used to form the corresponding subsystems along with the optimizations for the minimum discrepancy objective. Additionally, the parallel processing technique is adopted to save calculation time and to take full advantage of the module autonomy.

In addition to the two original design variables P_c and r_c, eight interdisciplinary coupling variables were added in the system-level optimization to compose the design variable vector X for the CO formulation: the thrust-chamber thrust at altitude F_1, the fuel-pump-outlet pressure P_f, the oxidizer-pump-outlet pressure P_o, the gas-generator pressure P_3, the thrust-chamber thrust F_c, the turbo-exhaust-gas thrust F_t, the chamber mass flow rate q_1, and the gas-generator mass flow rate q_3. The system-level optimization objective is to maximize I_e and N_e, which is the same as the original problem objective.

In the thrust-chamber subsystem, the design variables are P_c, r_c, and F_1. In the gas-generator subsystem, the design variables are P_c, P_f, and P_o. In the turbopump subsystem, the design variables are P_c, P_f, P_o, P_3, r_c, and q_1. The subsystem optimization objectives are to minimize the discrepancies between target variables and corresponding subsystem values while satisfying the local constraints.

The general convergence of the CO formulation has not yet been completely demonstrated. In some cases, applying CO can successfully converge to an optimal solution [23,24], whereas in other cases, the convergences are not ideal [25–28]. One way to improve the convergence performance is to relax system-level compatibility constraints: that is, change compatibility constraints from equality to inequality constraints [26,28]. Therefore, the CO formulation of system-level optimization is as follows:

$$\begin{gathered}\text{find } X\\ \min -\omega_1 I_e/A - \omega_2 N_e/B\\ \text{subject to } J_1 \le \varepsilon_1\\ J_2 \le \varepsilon_2\\ J_3 \le \varepsilon_3\\ I_e \ge I_{e0}\\ F_0 \le F \le F_1\\ r_{e0} \le r_e \le r_{e1}\end{gathered} \tag{7}$$

where J_1, J_2, and J_3 are quadratic compatibility constraints, one for each subsystem having the form

$$J_i = \sum_j (x_j - x_j^*)^2 \tag{8}$$

where x_j^* is the solution of the subsystem optimization problem, and the relaxation factors ε_1, ε_2, and ε_3 are set to 0.01 here.

Compared with the standard CO formulation, two improvements are incorporated in the present study to enhance the convergence performance and exploration ability:

1) The values of subsystem constraints are transferred to the system-level optimization as additional constraints, so that the system-level optimization problem will not only coordinate the relationships of the three subsystem optimizations, but also consider whether the assigned system-level variable values are reasonable for subsystem optimizations. This will lead to a quicker convergence of the system-level optimization.

2) Not all system-level design variables are transferred to each subsystem; only those used in the subsystem optimization are transferred to the corresponding subsystem as design targets. Thus, the numbers of subsystem design variables are different from each other, which will reduce the dimensions of subsystem optimization problems.

The CO results for the aforementioned specific example are tabulated in Table 3. Since the initial values of design variables are given randomly, the engine specific impulse and vacuum thrust-to-weight ratio cannot be determined at the initial point. After optimization, all design variables, including coupling variables, are set to proper values, and an optimum solution is achieved as well. The values of system parameters and objectives obtained by applying the CO formulation are slightly different from those obtained through the MDF solution, due to the convergence error of compatibility constraints. As for the total execution time, the CO formulation, using the hybrid algorithm at system-level optimization and the gradient-based algorithm at local optimization, takes about 3 h, which is three times longer than MDF takes to reach an optimal solution in this case.

F. Multi-Objective Optimization

Multi-objective optimization seeks to determine an optimum design that involves the minimization (or maximization) of multiple-objective functions, which are usually in conflict with each other. It is difficult to optimize all the objectives in an optimization problem. Comparing objectives directly is prohibited, since they usually have different measurement units and magnitudes. The expression of the optimal solution cannot be specified mathematically, as the designers may have different demands on each of objectives.

Various methods of multi-objective processing may find different solutions to the same problem. All of these solutions are noninferior solutions, which are also known as Pareto-frontier solutions. Since the utopian solution cannot be achieved in the multi-objective optimization, designers usually choose the optimal compromise solution from the Pareto-frontier solutions according to their own preferences.

For the hydrogen–oxygen rocket engine example, the variable-weight method, e-constraint method [29], and neighborhood cultivation genetic algorithm (NCGA) [30] are adopted to obtain the Pareto-frontier solutions to the maximization of the engine specific impulse and vacuum thrust-to-weight ratio.

In the variable-weight method, eight groups of symmetrical weight factors are used to construct the evaluation function. Figure 2 shows the Pareto frontier obtained by this method, wherein ω_1 is the weight of the vacuum thrust-to-weight ratio, and ω_2 is the weight of

Table 3 Optimization solution from CO formulation

	System-level design variables										Objectives	
Case	P_c, MPa	r_c	P_f, MPa	P_o, MPa	P_3, MPa	F_1, kN	F_c, kN	F_t, kN	q_1 kg/s	q_3 kg/s	I_e s	N_e N/kg
Initial values	11.0	6.0	18.0	15.0	9.0	665.0	697.0	2.0	158.0	9.0	——	——
Optimization values	8.0	5.3	13.3	11.2	6.9	664.7	698.7	1.4	156.7	6.0	439.8	670.6

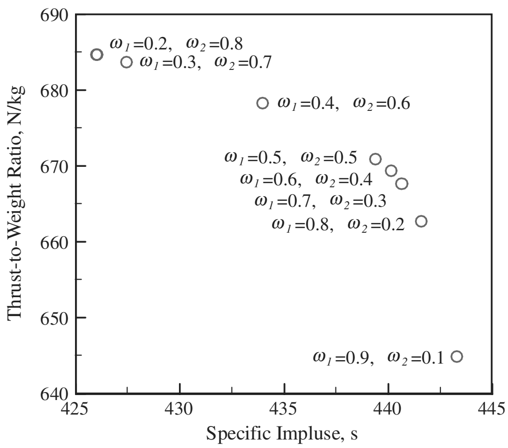

Fig. 2 Pareto frontier from variable-weight method.

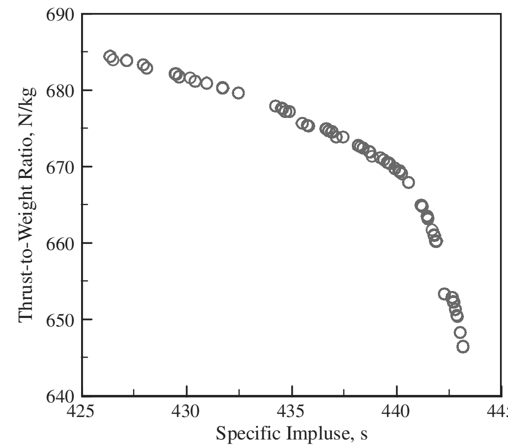

Fig. 4 Pareto frontier from NCGA method.

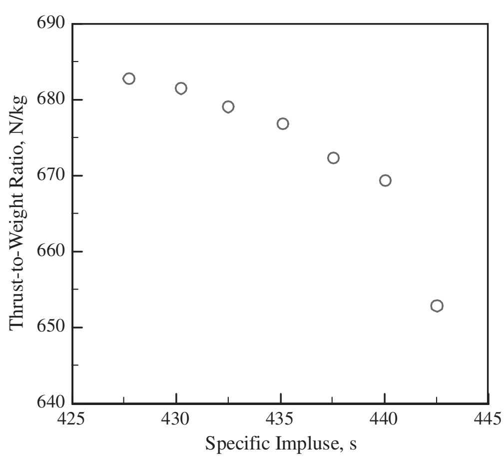

Fig. 3 Pareto frontier from *e*-constraint method.

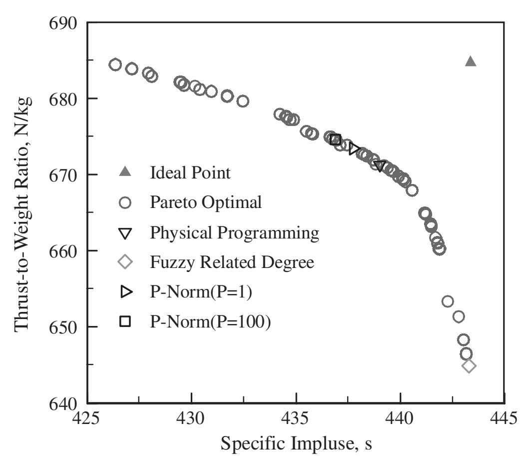

Fig. 5 Results from different methods.

the engine specific impulse. It can be found that evenly changing the weights does not guarantee the even distribution of the corresponding solution on the Pareto frontier.

In the e-constraint method, the objective function is defined to maximize the vacuum thrust-to-weight ratio as a single object. Meanwhile, the engine specific impulse is formatted as a constraint, with its upper limit and lower limit confirmed to be 442.5 s and 427.5 s, respectively, which is defined using the multicriteria tradeoff analysis approach. Figure 3 shows the Pareto frontier obtained by the e-constraint method, wherein the Pareto-frontier solutions are distributed uniformly, as the value of engine specific impulse constraint is defined evenly. Thus, the e-constraint method can result in controllable Pareto-frontier solutions.

In the NCGA method, the generation number is set to 50, the population size is 10, the crossover rate is 1.0, and the mutation rate is 0.01. Figure 4 shows the Pareto frontier obtained by NCGA. Compared with the other two methods, NCGA can find the Pareto-frontier solutions more effectively and efficiently with respect to the exploration domain and computational consumption.

Pareto-frontier domains obtained from the above three methods are different, but the optimization results of the test problem are approximately consistent in the superposition area. Based on the Pareto-frontier solutions, methods of ideal point with p-norm [29], fuzzy related degree [31], and physical programming [32] are used to acquire optimal compromise solutions. The results are shown in Table 4.

For this specific cryogenic-propellant rocket engine, the single-objective optimization value of engine specific impulse is $I_e^* = 443.4$ s, and vacuum thrust-to-weight ratio is $N_e^* = 684.7$ N/kg. Figure 5 shows the results of all multi-objective optimization methods and all multi-objective decisions, which could provide a reference for engine designers. Note that different multi-objective decisions have distinct optimal compromise solutions according to different mathematical theories, but they all lie on the Pareto frontier. Although there is no priority of the weight factors of the engine specific impulse and vacuum thrust-to-weight ratio in all the methods of multi-objective processing, each of the objectives is not optimized to the same degree, due to the intrinsic characteristics of different methods.

IV. Reliability-Based LRE System Analysis and Optimization

In deterministic optimization, the designs are often driven to the limit of the constraints (also known as the active constraints at the optimum). These designs may be subject to failure due to inherent

Table 4 Optimal compromise solutions from different methods

Case	Ideal point method with P-norm ($P = 1$)	Ideal point method with P-norm ($P = 100$)	Fuzzy related degree method	Physical programming method
I_e, s	437.7	436.9	443.3	439.0
N_e, N/kg	673.4	674.6	644.9	671.3

Table 5 Deterministic and reliability-based optimization solutions

Case	P_c		r_c			Reliability	
	μ, MPa	σ, MPa	μ, MPa	σ, MPa	I_e s	$F_0 \leq F \leq F_1$	$r_{e0} \leq r_e \leq r_{e1}$
Deterministic optimization solution (MDF)	8.0	0.0264	5.30	0.0175	439.4	0.9998	0.498
Reliability-based optimization solution	8.0	0.0264	5.36	0.0177	438.0	1	1

uncertainties, which exist both in the mathematical modeling and simulation tools, and due to the variability in physical quantities of the manufacture. The existence of physical uncertainties and model uncertainties require a reliability-based design analysis and optimization (RBDA&O) to be taken into account. Many researchers have focused on RBDA&O, and more detailed information can be found in [33,34].

When the LRE is running in practical application, parameters such as the thrust-chamber pressure and the mixture ratio cannot be a fixed value. Usually, they will fluctuate within a range. Will the LRE performance meet the design demand when parameters fluctuate? If so, what is the reliability? To solve these questions, a preliminary RBDA&O was performed on the design of the engine system parameters addressed herein.

For the hydrogen–oxygen rocket engine case, two random variables are defined: thrust-chamber pressure and mixture ratio, which are also design variables in deterministic optimization. It is assumed that the random variable variation follows a normal distribution, and both of the variation coefficients, which are equal to the value of the standard deviation σ divided by the mean μ for a specified random variable, are set to 0.33%. This indicates that thrust-chamber pressure can vary by 0.02–0.03 MPa, and the mixture ratio can vary by 0.01–0.02.

Based on the above assumption, the RBDA is first executed on one of the deterministic design optimum points, and the corresponding results are shown in Table 5. The reliability of the engine thrust is 0.9998. But the reliability of the engine mixture ratio is 0.498, close to its lower bound r_{e0}, which is too low to satisfy the design demand. Thus, the RBDO should be performed to improve the design quality.

After the RBDO using the feasible directions algorithm, the LRE system-design-quality level, especially the reliability of mixture ratio, has increased. In the assumed variation range, the reliabilities of the engine thrust and mixture ratio both reach 100%. Table 5 shows that the engine specific impulse has decreased a little with the increase in reliability, which means there is a tradeoff between the reliability and the LRE performance.

V. Conclusions

A general design optimization framework has been developed to select the best system parameters for LREs with gas-generator cycles. MDO formulations such as MDF and CO are employed, evaluated, and compared in the framework. Several methods dealing with multi-objective problems, resulting in Pareto-frontier solutions or optimal compromise solutions, are also provided for engine designers. To obtain the global optimal solution, a hybrid optimization method combining a genetic algorithm and sequential quadratic programming is used. The system design of a particular hydrogen–oxygen rocket engine is improved after optimization; for instance, the specific impulse increased by 2.5% and the vacuum thrust-to-weight ratio increased by 4.5% using the fuzzy related degree method. In addition, the framework can execute primary RBDA&O to enhance the LRE design reliability.

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